1(CCE.M)2

Statistics—I

(23)

Time : Three Hours] [Maximum Marks : 300

INSTRUCTIONS

(i) Answers must be written in English.

(ii) The number of marks carried by each question is indicated at the end of the question.

(iii) The answer to each question or part thereof should begin on a fresh page.

(iv) Your answer should be precise and coherent.

(v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.

(vi) Candidates should attempt any **five** questions, choosing at most **two** from each Section.

(vii) If you encounter any typographical error, please read it as it appears in the text-book.

(viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.

(ix) No continuation sheets shall be provided to any candidate under any circumstances.

(x) Candidates shall put a cross (×) on blank pages of Answer Script.

3. (a) Define convergence in probability and convergence almost surely. Show that the sample mean of a normal $N(\mu,\sigma^2)$ distribution converges in probability to μ. 30

(b) Define weak law of large numbers (WLLN) and strong law of large numbers (SLLN). Examine for what values of α, sequence $\{X_k\}$ with

$$P\,(X_k = \pm K^{\alpha}) = \frac{1}{2}\ ,$$

WLLN/SLLN hold. 30

4. (a) Let the pdf of a random variable X be

$$f(x) = \frac{\lambda}{\pi}.\frac{1}{\lambda^2 + (x-\mu)^2}, -\infty < x < \infty$$

Obtain characteric function (CF) of X. 30

(b) Let CF of a random variable X be $\phi(t) = \exp\left[i\mu t - \frac{\sigma^2 t^2}{2}\right]$

Obtain pdf of x. 30

SECTION–II

(Statistical Inference)

5. (a) Let X has uniform $U(0, \theta)$ distribution, show that $X_{(n)} = \max(x_1, \ldots, x_n)$ is complete and sufficient statistic for θ, where $x_1, \ldots .x_n$ is a random sample from $U(0, \theta)$. 40

(b) Show that consistent estimator may not be unbiased. 20

6. (a) Show that maximum likelihood estimator (mle) may not be unique. 20

(b) Obtain uniformly minimum variance unbiased estimator (umvue) for θ^2 from normal $N(\theta, \sigma^2)$ distribution, when

(i) σ^2 is known and

(ii) σ^2 is unknown. 40

7. (a) Define monotone likelihood ratio (MLR) family of distributions.

Show that $f(x, \theta) = \begin{cases} \theta x^{\theta-1} & , \quad 0 < x < 1 \\ 0 & , \quad \text{else where} \end{cases}$

is a member of MLR family of distributions.
Obtain UMP test of size α for testing.

$H_0 : \theta \le \theta_0$

$H_1 : \theta > \theta_0$ 30

(b) Explain likelihood ratio test (LRT) method for finding the critical region.

Obtain UMP test of size α for testing.

$H_0 : p = p_0$

$H_1 : p > p_0$

Using LRT method, on the basis of a random sample of size n from Bernoulli B(1, p) distribution. 30

8. (a) Prove that :

$$P[X_{(r)} < \xi_p < X_{(s)}] = \sum_{i=r}^{s-1} \binom{n}{i} p^i (1-p)^{n-i}$$

where ξ_p is the p-th quantile of the continuous distribution F(x) and X(r) is the r-th order statistic from a random sample of size n from F(x). 30

(b) What do you mean by 'run' ? Explain. Describe a test of randomness based on the total number of runs. 30

(xi) No blank page be left in between answer to various questions.

(xii) No programmable Calculator is allowed.

(xiii) No stencil (with different markings) is allowed.

SECTION—I

(Probability)

1. (a) Let X be a non-negative continuous random variable with the distribution function F(x), then show that

$$E(x) = \int_0^\infty [1 - F(x)]\,dx \qquad 20$$

(b) Let f(x, y) = c, $0 < x < y < 1$

0, otherwise

be the joint probability density function (pdf) of (X, Y)

(i) Obtain c

(ii) Obtain conditional pdf of X given Y = y and Y given X=x. Hence obtain E[X|Y=y] and E[Y|X=x]. 40

2. (a) Let Y_1 denote the first order statistic of a random sample of size n from a distribution that has the pdf

$$f(x) = \begin{cases} e^{-(x-\theta)} &, \quad \theta < x < \infty \\ 0 &, \quad \text{otherwise} \end{cases}$$

Obtain the limiting distribution of $Z_n = n\,(Y_1 - \theta)$ 30

(b) State Chebyshev's inequality. Let X be a random variable, such that

$E(x) = 3$ and $E(x^2) = 13$. Use Chebyshev's inequality to determine lower bound of $P[-2 < X < 8]$. 30

SECTION–III

(Linear Inference and Multivariate Analysis)

9. (a) In a linear regression model :

$Y_i = \beta_0 + \beta_1 x_i + e_i$, z = 1, 2, ... ,n

where β_0, β_1 are unknown parameters and e_i are independent normal random variables with $E(e_i) = 0$ and var $(e_i) = \sigma^2$,

Z = 1, 2, ... n, σ unknown.

Obtain 100 $(1-\alpha)$ % confidence interval for β_0. 30

(b) Let X_1, X_2, X_3 be three uncorrelated random variables having common variance σ^2.

If $E(X_1) = \theta_1 + \theta_2$

$E(X_2) = 2\theta_1 + \theta_2$

$E(X_3) = \theta_1 + 2\theta_2$

Obtain least square estimator of θ_1 and θ_2 and show that they are unbiased. 30

10. (a) Let X has multivariate normal $N_p(\mu, \Sigma)$ distribution. Develop the test statistic to test $H_0 : \mu_1 = \mu_2 = ... = \mu_p$ 30

(b) Let X be a p-variate normal with mean vector $\underline{0}$ and dispersion matrix Σ. Obtain multiple correlation between $(X_1, \underline{X}_2)$, where $\underline{X}_2 = \text{col}\,(X_2, ..., X_p)$. 30

11. Discuss two way analysis with one observation per cell and obtain analysis of variance (ANAVOA) table to test the hypothesis of interest. 60

12. Write explanatory notes on :

(a) Mahalanobis D^2 30

(b) Orthogonal Polynomial. 30