MATHEMATICS

SYLLABUS

1. Real Analysis

Elementary Set Theory, Countable and uncountable sets, real number system as a complete ordered field, Archimedean property, supremum and infimum, sequences and series, convergence and absolute convergence. Various types of convergences, limit superior and limit inferior. Bolzano-Weierstrass theorem, Heine-Borel theorem. Continuity and uniform continuity, differentiability, sequences and series of functions and their convergence viz. Cauchy's Criteria and Weierstrass M-test for convergences. Riemann integration of bounded functions and its various properties, monotone functions. Types of discontinuities, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivatives, partial derivative. Total derivative, mean value theorem for differentiable functions.

2. COMPLEX ANALYSIS

Algebra of Complex numbers, complex planes, functions of a complex variable. Continuity, differentiability, CR-equations, analytic functions. Necessary and sufficient condition for analyticity, harmonic functions, harmonic Conjugate, contour integration. Cauchy integral theorem, Cauchy's Integral Formula, Liouville's theorem, maximum modulus principle, fundamental Theorem of algebra. Morera's theorem, Schwarz lemma, open mapping theorem, Taylor series expansion. Elementary Linear Transformations, Mobius transformation, conformal mapping, singularities and its types, Riemann's theorem on removable singularities, Laurent's series expansion, Hurwitz theorem, Cauchy residue theorem, integrals of rational and trigonometric functions by residue theorem.

3. TOPOLOGY

Definition of a metric space, Examples, Open and closed sets, compactness in metric spaces, continuity, uniform continuity, complete metric spaces, Topological spaces, Definition and examples, Elementary Properties, Interior, exterior and boundary of a set, closure of a set, Kuratowski Axioms, Neighbourhood system, Local base and base, subspaces and relative topology, continuous maps, open maps, closed maps and their characterization, Homeomorphism, Separation Axioms, Lebesgue Covering lemma. Product topology, weak topology, compactness and connectedness in topological spaces, Tychnoff's theorem.

4. FUNCTIONAL ANALYSIS

Contraction mapping, Banach Contraction principle, Application to differential and integral equation, Baire's category theorem and its application, Arzela-Ascoli theorem, Normed linear spaces, Banach space, Finite dimensional normed linear spaces, equivalenceof norms, Quotient spaces, Riesz lemma, Bounded linear operators, dual

spaces, Hahn-Banach theorem and its application, uniform boundedness principles, open mapping theorem, Bounded inverse theorem, Closed graph theorem, Inner product spaces, Cauchy-Schwarz inequality, Pythagorean theorem, Hilbert spaces, Orthogonal complements and Direct sums, Orthonormal sets, projection theorem, Bessel's inequality, Riez Representation theorem.

5. ABSTACT ALGEBRA

Groups, Subgroups, Normal subgroups, Quotient groups Homomorphism, Cyclic groups, Permutation and permutation groups, Cayley's theorem, Class equation, Sylow's theorem, Rings, Ideals, Prime and maximal ideals, Integral domain, Polynomial rings, Eisenstein irreducibility criteria, Fields, Finite fields, Field extensions, Finite modules, Submodules, Cyclic modules, Finite direct sum and product of modules, Finitely generated modules, Quotient modules, Homomorphism, Fundamental theorem, Isomorphism theorem, ACC & DCC, Schur's, Lemma.

6. LINEAR ALGEBRA

Vector spaces, Subspaces, Linear dependence and independence, Basis, Dimension of vector spaces, Linear transformation, Algebra of linear transformations, Matrices, Algebra of Matrices, Rank and determinant of Matrices, Linear equation and solutions, Eigen values and vectors, Cayley-Hamilton Theorem, Matrix representation of a linear transformation, Change of basis, Canonical form, Diagonal forms Triangular form, Jordan forms, cyclic subspaces, Rational canonical form, quotient spaces, bilinear forms, Alternating bilinear form, Symmetric bilinear form, Quadratic form.

7. Differential Equations

First order ODE, linear differential equation with constant coefficients of order n, linear dependence and independence of solution, Wronskian, Method of reduction of order, Method of variation of parameters, power series solution about ordinary points, solution of Legendre equation, Legendre polynomial, Rodrigues Formula, solution of Bessel's equation, Simultaneous differential equations, Total differential equations, Lagrange's and Charpits method of solving PDE's, Cauchy problem for first order PDE's, Classification of 2^{nd} order PDE's, Heat and Wave equation.

8. THEORY OF NUMBER

Divisibility, division algorithm, GCD, prime numbers and related theorems. Fundamental theorem of arithmetic, standard form of an integers, linear Diophantine equations. Necessary and sufficient condition for solution of linear Diophantine equations. Radix representation theorem. Euclid's theorem, infinitude of primes, congruences and properties, CRS and RRS, multiplicative functions, Euler's Totient function, Fermat's little theorem, Chinese remainder theorem, Wilson's theorem and applications.

Order of an element, primitive roots. Quadratic and non-residues. Legendre symbols and Jacobi symbols and their properties. quadratic reciprocity laws, number theoretic functions

viz. $\phi(n)$, $\tau(n)$, $\sigma(n)$ and $\mu(n)$. Perfect numbers, Mobius function, and formula, Farrey fractions.

9. NUMERICAL METHODS

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson Method, Rate of Convergence, Solutions of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel Methods, Finite differences, Lagrange, Hermite and Spline interpolation, Numerical differentiation and integration, Numerical solutions of ordinary differential equations using Picard, Euler, modified Euler and Runge-Kutta Methods.

10. DIFFERENTIAL GEOMETRY

Curves, differentiable curves, arc length, parameterization by arc length, plane curvature, directed curvature, fundamental theorem for place curves, centre of curvature. Curves in space, tangent, normal and binormal unit vectors, curvature and torsion, Serret Frenet frame. Properties of curves like: Helix, Bertrand-Mete, involute & evolute, curves on spheres. Regular surfaces, co-ordinate charts, differentiable function, Diffeomorphism, tangent plane, unit normal vector, oriented surfaces, angle between two curves, orthogonal parameterization, area and curvature for surfaces. Euler's work on surfaces, principal curvature, line of curvature, Rodrigues Formula, Gauss map, second fundamental form, Meusnier's theorem.