Total No. of Printed Pages : 7 **Roll No. ....**

1[CCE.M]1

Mathematics–I

(15)

Time : Three Hours Maximum Marks : 300

INSTRUCTIONS

(i) Answers must be written in English.

(ii) The number of marks carried by each question is indicated at the end of the question.

(iii) The answer to each question or part thereof should begin on a fresh page.

(iv) Your answers should be precise and coherent.

(v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.

(vi) Candidates should attempt any **five** questions.

(vii) If you encounter any typographical error, please read it as it appears in the text book.

(viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.

11. (a) If a point moves with constant acceleration, the space average of the velocity over any distance is

$$\frac{2}{3}\frac{u_1^2 + u_1 u_2 + u_2^2}{u_1 + u_2}$$

where u_1 and u_2 are initial and final velocities. Is this greater or less than the time average ? 20

(b) A particle is projected with velocity $2\sqrt{ag}$ so that it just clears two walls of equal heights a, which are at a distance 2a from each other, show that the latus rectum of the path is 2a and the time of passing between the walls is $2\sqrt{\frac{a}{g}}$. 20

(c) A body moving in a straight line OAB with S.H.M has zero velocity at the points A and B whose distances from o are a and b respectively, and has a velocity *v* when half way between them. Show that complete period is $\pi(b - a)/v$. 20

12. (a) Find the equation of Catenary of uniform length. 20

(b) A thin hemispherical bowl, of radius b and height W rests in equilibrium on the highest point of a fixed sphere of radius a which is rough enough to prevent any sliding, inside the bowl is placed a small smooth sphere of weight w. Show that the equilibrium is unstable unless $w > W\frac{(a-b)}{2b}$. 20

3. (a) Evaluate $\lim_{x\to\infty} \frac{x(\log x)^3}{1+x+x^2}$. 15

(b) Express the function $2x^3 + 7x^2 + x - 1 = 0$ in powers of $(x - 2)$. 20

(c) If $u = x^3 y^2 \sin^{-1}(y/x)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 5u$ and

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 20u.$$ 25

4. (a) Show that the semi-vertical angle of a right cone of given total surface (including area of base) and maximum volume is $\sin^{-1}(1/3)$. 20

(b) Show that the normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at P(0, –3) meets the curve again at two points. Find the equation of the tangents to the curve at these points. 20

(c) Trace the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$ or $x^2(x^2 + y^2) = a^2(x^2 - y^2)$ (these curves are one and the same). 20

5. (a) Evaluate $\int_0^1 \frac{x^{2n}}{\sqrt{1-x^2}}dx$ and hence find the sum of the series $\frac{1}{2n+1} + \frac{1}{2}\cdot\frac{1}{2n+3} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2n+5} + \text{........to}\infty$

$$= \frac{(2n-1)(2n-3).....3\cdot 1}{2n(2n-1)..........4\cdot 2}\cdot\frac{\pi}{2}.$$ 20

(b) Find the area bounded by the curve $y^2(a + x) = (a - x)^3$ and its asymptote. 20

(c) Find the centre of gravity of the area between the curve $y = c \cosh (x/c)$, the coordinate axes and the ordinate $x = a$. 20

6. (a) Show that the lines $7x - 4y + 7z + 16 = 0 = 4x + 3y - 2z + 3$ and $x - 3y + 4z + 6 = 0 = x - y + z + 1$ are coplanar and find the plane containing the lines. 30

(b) Show that the lines $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$ and

$x + 2y + 3z - 8 = 0 = 2x + 3y + 4z = -11$ are intersecting. Find the point of their intersection and the equation to the plane containing them. 30

7. (a) Find the equation to the right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 25

(b) Show that the locus of the point for which the chord of contact with respect to $y^2 = 4ax$ subtends a right angle at the vertex of the parabola is $x + 4a = 0$. 20

(c) Find the condition that the plane $lx + my + nz = p$ should touch the conicoid $ax^2 + by^2 + cz^2 = 1$. 15

8. Solve the following differential equations :

(a) $(2x - y + 1)\, dx + (2y - x - 1)\, dy = 0$.

(b) Find Orthogonal trajectories of hyperbolas $xy = c^2$.

(c) Solve $(x^2D^2 - x D + 2)\, y = x \log x$.

(d) Use Convolution theorem to find the Laplace inverse of $\frac{1}{s(s+1)(s+2)}$. 15 each

9. (a) If $(\vec{a}, \vec{b}, \vec{c})$ and are reciprocal triads of vectors, show that $\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}' = 0$. 20

(b) Show that . 20

(c) If $\vec{r}(t) = 5t^2 i + t\, j - t^3 k$, prove that is equal to $-14i + 75j - 15k$. 20

10. (a) Prove that the tangent at any point on the curve whose equations are $x = 3t$, $y = t^2$, $z = 2t^3$, makes a constant angle with the line $y = z - x = 0$. 20

(b) Show that the necessary and sufficient condition for the curve to be a plane curve is $[r' \; r'' \; r''']= 0$. 20

(c) State and prove Gauss theorem. 20

(ix) No continuation sheets shall be provided to any candidate under any circumstances.

(x) Candidates shall put a cross (X) on blank pages of Answer Script.

(xi) No blank page be left in between answer to various questions.

1. (a) Show that a set of vectors which contains at least one zero vector is linearly dependent. 30

(b) If $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ verify Cayley-Hamilton theorem. Hence find A^{-1}. 30

2. (a) Find the eigen values of the matrix :

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$ 30

(b) Determine the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$ 30

(c) A body rests in equilibrium on a rough inclined plane. The inclination of the plane is gradually increased, till it is α, when the body begins to slide down. Show that α is the angle of friction. 20