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**COMBINED COMPETITIVE (PRELIMINARY) EXAMINATION, 2010**

Serial No.

**MATHEMATICS**

**Code No. 13**



*Time Allowed : Two Hours*

*Maximum Marks : 300*

**INSTRUCTIONS**

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC, IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C OR D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE RESPONSE SHEET.
3. You, have to enter your Roll Number on this  Your Roll No.  
Test Booklet in the Box provided alongside.  
*DO NOT* write *anything* else on the Test Booklet.
4. This Booklet contains 100 items (questions). Each item comprises *four* responses (answers). You will select *one* response which you want to mark on the Response Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.
5. In case you find any discrepancy in this test booklet in any question(s) or the Responses, a written representation explaining the details of such alleged discrepancy, be submitted within three days, indicating the Question No(s) and the Test Booklet Series, in which the discrepancy is alleged. Representation not received within time shall not be entertained at all.
6. You have to mark all your responses **ONLY** on the separate Response Sheet provided. *See directions in the Response Sheet.*
7. All items carry equal marks. Attempt **ALL** items. Your total marks will depend only on the number of correct responses marked by you in the Response Sheet.
8. Before you proceed to mark in the Response Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Response Sheet as per instructions sent to you with your Admit Card and Instructions.
9. While writing Centre, Subject and Roll No. on the top of the Response Sheet in appropriate boxes use **“ONLY BALL POINT PEN”**.
10. After you have completed filling in all your responses on the Response Sheet and the examination has concluded, you should hand over to the Invigilator only the Response Sheet. You are permitted to take away with you the Test Booklet.

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## **ROUGH WORK**

1. Rank of the matrix  $\begin{bmatrix} 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \\ 10 & 11 & 12 & 13 \\ 15 & 16 & 17 & 18 \end{bmatrix}$

- (A) 4 (B) 3  
(C) 2 (D) 1

2. Test whether the matrix equation is consistent? If so, solution is .....

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & -5 & 5 \\ 3 & 9 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 4 \end{bmatrix}$$

- (A) Inconsistent (B) Consistent,  $x = -1, y = 1, z = 2$   
(C) No solution, exist (D) Consistent,  $x = 1, y = 1, z = 0$

3. Inverse of the matrix  $\begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$

- (A)  $\begin{bmatrix} 6 & -3 \\ -2 & 7 \end{bmatrix}$  (B)  $\frac{1}{36} \begin{bmatrix} -6 & 3 \\ 2 & -7 \end{bmatrix}$   
(C)  $\frac{1}{36} \begin{bmatrix} 6 & 3 \\ -2 & -7 \end{bmatrix}$  (D)  $\frac{1}{36} \begin{bmatrix} 6 & -3 \\ -2 & 7 \end{bmatrix}$

4. If  $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$  then  $A^4 = \dots\dots\dots ?$

- (A)  $\begin{bmatrix} 976 & 960 \\ 320 & 336 \end{bmatrix}$  (B)  $\begin{bmatrix} 5^4 & 3^4 \\ 1 & 3^4 \end{bmatrix}$   
(C)  $\begin{bmatrix} 976 & 320 \\ 336 & 960 \end{bmatrix}$  (D)  $\begin{bmatrix} 976 & 336 \\ 320 & 960 \end{bmatrix}$

5. Value of the determinant  $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 8 & 0 & 2 \end{bmatrix}$

- (A) zero (B) 8  
(C) -8 (D) 16

6. Limit of  $\frac{x^n - a^n}{x - a}$ , as  $x \rightarrow a$

- (A) zero (B)  $\left(\frac{0}{0}\right)$   
(C)  $n a^{n-1}$  (D)  $x^{n-1} \times a$

7. Taylor's series of  $\tan^{-1} x$ , in powers of  $x$  :

- (A)  $x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$  (B)  $1 + x + x^2 + \dots$   
(C)  $x - \frac{x^3}{3} + \dots$  (D)  $x + \frac{x^3}{3} + \dots$

8. Differentiation of  $a^x$ , w.r.t.  $x$ , is :

- (A)  $a^x$  (B)  $a^{x-1}$   
(C)  $a^x \log a$  (D)  $x a^{x-1}$

9. Split 24 into 3 parts such that the continued product of the first, square of the second and cube of the third may be minimum.

- (A) 4, 8, 12 (B) 4, 6, 14  
(C) 3, 9, 12 (D) 3, 16, 5

10. The radius of curvature of  $y = e^x$  at  $x = 0$  is :

- (A)  $\sqrt{2}$  (B) 2  
(C)  $(2)^{1/4}$  (D)  $(2)^{3/2}$

11. The asymptotes of the curve  $ay^2 = \left(\frac{x}{x^2 - a^2}\right)$  are :

- (A)  $x = 0, a$  (B)  $x = a; -a$   
(C)  $x = 0, -a$  (D)  $a = 0; x = 0$

12. Curvature of the circle  $x^2 + y^2 - 5x + 6y - 16 = 0$  is :  
 (A)  $(5)^{-3/2}$  (B)  $(5)^{3/2}$   
 (C) 15 (D)  $\sqrt{15}$
13. Equation of the tangent at (3, 4) to the circle  $x^2 + y^2 = 25$  :  
 (A)  $x + y + 5 = 0$  (B)  $3x + 4y + 25 = 0$   
 (C)  $3x + 4y - 25 = 0$  (D)  $3x + 4y = 0$
14. Equation of the normal to the parabola  $y^2 = 4x$  at (4, 4) is :  
 (A)  $2x + y - 12 = 0$  (B)  $2x + y + 4 = 0$   
 (C)  $2x + y - 4 = 0$  (D)  $2x - 4y + 8 = 0$
15. Points of inflexion are :  
 (A) Maximum points (B) Minimum points  
 (C) Neither Maximum nor Minimum (D) Double points
16. The value of  $\int_0^{\pi/2} \cos^7 \theta \, d\theta =$   
 (A)  $\frac{48}{105}$  (B)  $\frac{105}{48}$   
 (C)  $\frac{48\pi}{105}$  (D)  $\frac{48\pi}{210}$
17. The value of  $\int_0^{\pi} \frac{d\theta}{3 + 2 \cos \theta} =$   
 (A)  $\frac{\pi}{2}$  (B)  $\frac{\pi}{2\sqrt{2}}$   
 (C)  $\frac{\pi}{5\sqrt{2}}$  (D)  $\frac{\pi}{2\sqrt{5}}$
18. Volume generated by solid of revolution of  $y^2 = 4ax$  about x axis by its Latus rectum; is :  
 (A)  $\pi a^3$  (B)  $(\pi a)^3$   
 (C)  $\frac{4}{3} \pi a^3$  (D)  $2\pi a^3$

19. Surface area generated by revolving  $y = \sin x$ , between 0 and  $\pi$  about x axis is :
- (A)  $4\pi$  (B)  $\pi$   
 (C)  $2\pi$  (D)  $3\pi/2$
20. Area of the region bounded by the curve  $y = 3x^2 - x$ , and the x axis between  $x = -1$  and  $x = 1$  is :
- (A)  $27/55$  (B)  $55/27$   
 (C)  $5/27$  (D)  $55/7$
21. Value of  $\int_{-\pi/2}^{\pi/2} x \sin x \, dx$  is :
- (A) 0 (B) 1  
 (C) 2 (D) 3
22. If  $x^y + y^x = c$ , then  $(dy/dx) - (\text{partial diffn w.r.t. } x)/\text{p.d. w.r.t. 'y'}$  gives :
- (A)  $yx^{y-1}/xy^{x-1}$  (B)  $-(yx^{y-1} + y^x \log x)/(xy^{x-1} + x^y \log x)$   
 (C)  $-(yx^{y-1} + y^x \log x)$  (D)  $xy^{x-1} + x^y \log x$
23. If  $x^2 - y^2 + u^2 + 2v^2 = 1$  and  $x^2 + y^2 - u^2 - v^2 = 2$  then  $\frac{\partial u}{\partial x} = \dots\dots\dots?$  and  $\frac{\partial v}{\partial x} = \dots\dots\dots?$
- (A)  $3xu; -2xv$  (B)  $\frac{3x}{u}; \frac{-2x}{v}$   
 (C)  $\frac{3u}{x}; \frac{-2v}{y}$  (D)  $xu; -xv$
24. If  $u = \sin^{-1}\left(\frac{x+y}{xy}\right)$  then  $x\left(\frac{\partial u}{\partial x}\right) + y\left(\frac{\partial u}{\partial y}\right) = \dots\dots\dots?$
- (A)  $-\sin u$  (B)  $\tan u$   
 (C)  $\cot u$  (D)  $-\tan u$
25. Test the series for convergence  $\sqrt{1} + \sqrt{2} + \sqrt{3} + \dots\dots\dots \infty$ .
- (A) convergent (B) divergent  
 (C) absolute convergent (D) oscillatory
26. Test the convergence of  $\sum_1^{\infty} \frac{(n+1)}{\sqrt{n}(n+2)}$ .
- (A) convergent (B) divergent  
 (C) oscillatory (D) absolute convergent

27. Singular solution of  $y = x \left( \frac{dy}{dx} \right) + \frac{1}{(dy/dx)}$  is :
- (A)  $y^2 = x$  (B)  $y^2 = 4x$   
 (C)  $4y^2 = x$  (D)  $y = 4x^2$
28. Rewrite the equation  $p = \sin(y - px)$  in CLAIRAUT'S equation :
- (A)  $y = px + \sin^{-1} p$  (B)  $y = cx + \sin^{-1} c$   
 (C)  $y + px + \sin^{-1} p = 0$  (D)  $x = py + \sin^{-1} p$
29. Solve  $yp^2 + (x - y)p - x = 0$ , here  $p = \frac{dy}{dx}$ .
- (A)  $(y + x - c)(x^2 + y^2 - c) = 0$  (B)  $(y - x - c)(x^2 + y^2 - c) = 0$   
 (C)  $y + x = x^2 + y^2$  (D)  $(y + x) = \phi(x^2 + y^2)$
30. Solution of  $\left( \frac{dy}{dx} \right)^2 - 8 \left( \frac{dy}{dx} \right) + 15 = 0$  is :
- (A)  $(y - 3x + c)(y - 5x + c) = 0$  (B)  $(y + 3x + c)(y + 5x + c) = 0$   
 (C)  $(y + 3x)(y + 5x) = 0$  (D)  $(y - 3x)(y - 5x) = 0$
31. Particular solution of  $(D^2 + 1)^2 y = x^4$ , is :
- (A)  $x^4 + 12x^2$  (B)  $x^4 + 24x^2$   
 (C)  $x^4 - 24x^2 + 72$  (D)  $24x$
32. In which the region of the curve  $y^2(a + x) = x^2(3a - x)$  does not lie ?
- (A)  $x > 0$   
 (B)  $0 < x < 3a$   
 (C)  $x \leq -a$  and  $x > 3a$   
 (D)  $-a < x < 3a$
33. Particular integral of  $D^2 y = x \sin x$  :
- (A)  $(Ax + B)$  (B)  $(1 - x) \sin x$   
 (C)  $(x + 1) \cos x - \sin x$  (D)  $(1 - x) \sin x - \cos x$
34. The curve  $9y^2 = x^2(4 - x^2)$  is symmetrical about
- (A) y-axis  
 (B) x-axis  
 (C)  $y = x$   
 (D) both the axis

35. Equation of parabola whose focus is  $(2, -3)$  and its directrix  $2y - 3 = 0$  :
- (A)  $(y + 3)^2 = (x - 2)$  (B)  $(x - 2)^2 = (y + 3)$   
(C)  $4(x^2 - 4x) + 36y + 43 = 0$  (D)  $4x(x - 4) = 9(y - 3)$
36. The axis of the parabola  $y^2 - 2y + 8x - 23 = 0$  is :
- (A)  $y = -1$  (B)  $x = -3$   
(C)  $x = 3$  (D)  $y = 1$
37.  $16x^2 - 3y^2 - 32x - 12y - 44 = 0$  represents :
- (A) an ellipse (B) a circle  
(C) a parabola (D) a hyperbola
38. The line  $4x + 2y = c$ , is a tangent to the parabola  $y^2 = 16x$ , then  $c$  is equal to :
- (A)  $-1$  (B)  $-2$   
(C)  $4$  (D)  $-4$
39. The point of intersection of the tangent at  $t_1 = t$  and  $t_2 = 3t$  to the parabola  $y^2 = 8x$ , is :
- (A)  $(6t^2, 8t)$  (B)  $(8t, 6t^2)$   
(C)  $(t^2, 4t)$  (D)  $(4t, t^2)$
40. The length of the latus rectum of the parabola  $y^2 - 4x + 4y + 8 = 0$  is :
- (A)  $8$  (B)  $6$   
(C)  $4$  (D)  $2$
41. The directrix of the parabola  $y^2 = x + 4$ , is :
- (A)  $x = \frac{15}{4}$  (B)  $x = \frac{-15}{4}$   
(C)  $x = \frac{-17}{4}$  (D)  $x = \frac{17}{4}$
42. The length of the latus rectum of the parabola whose vertex is  $(2, -3)$  and the directrix  $x = 4$  is :
- (A)  $2$  (B)  $4$   
(C)  $6$  (D)  $8$
43. The focus of the parabola  $x^2 = 16y$ , is :
- (A)  $(4, 0)$  (B)  $(0, 4)$   
(C)  $(-4, 0)$  (D)  $(0, -4)$
44. The vertex of the parabola  $x^2 = 8y - 1$ , is :
- (A)  $\left(-\frac{1}{8}, 0\right)$  (B)  $\left(\frac{1}{8}, 0\right)$   
(C)  $\left(0, \frac{1}{8}\right)$  (D)  $\left(0, -\frac{1}{8}\right)$



45. The line  $2x + 3y + 9 = 0$  touches the parabola  $y^2 = 8x$  at the point :
- (A)  $(0, -3)$  (B)  $(2, 4)$   
 (C)  $\left(-6, \frac{9}{2}\right)$  (D)  $\left(\frac{9}{2}, -6\right)$
46. The tangents at the end of any focal chord to the parabola  $y^2 = 12x$ , intersect on the line :
- (A)  $x - 3 = 0$  (B)  $(x + 3) = 0$   
 (C)  $y + 3 = 0$  (D)  $y = 3$
47. The angle between the two tangents drawn from the point  $(-4, 4)$  to  $y^2 = 16x$  is :
- (A)  $45^\circ$  (B)  $30^\circ$   
 (C)  $60^\circ$  (D)  $90^\circ$
48. The equation of a sphere with centre at  $(2, 3, 5)$  which touches the  $x_o y$  plane, is
- (A)  $(x + 2)^2 + (y + 3)^2 + (z + 5)^2 = 5^2$   
 (B)  $(x - 2)^2 + (y - 3)^2 + (z - 5)^2 = 5^2$   
 (C)  $x^2 + y^2 + z^2 + 4x + 6y + 10z + 63 = 0$   
 (D)  $x^2 + y^2 + z^2 = 5^2$
49. The length of the semi-major and the length of minor axis of the ellipse  $\frac{x^2}{144} + \frac{y^2}{169} = 1$  are :
- (A) 26; 12 (B) 13; 24  
 (C) 12; 26 (D) 13; 12
50. The distance between the foci of the ellipse  $9x^2 + 5y^2 = 180$ , is :
- (A) 4 (B) 6  
 (C) 8 (D) 2
51. The length of major and semi-minor axes of an ellipse are 8, 2 and their corresponding equation are  $y = 6$  and  $x = -4$ , then the equation of ellipse is :
- (A)  $\frac{(x + 4)^2}{4} + \frac{(y - 6)^2}{16} = 1$  (B)  $\frac{(x + 4)^2}{16} + \frac{(y - 6)^2}{4} = 1$   
 (C)  $\frac{(x + 4)^2}{16} - \frac{(y - 6)^2}{4} = 1$  (D)  $\frac{(x + 4)^2}{4} - \frac{(y - 6)^2}{16} = 1$
52. The straight line  $2x - y + c = 0$  is a tangent to the ellipse  $4x^2 + 8y^2 = 32$ , if  $c$  is :
- (A)  $\pm 2\sqrt{3}$  (B)  $\pm 6$   
 (C) 36 (D)  $\pm 4$

53. The sum of the distance of any point on the ellipse  $4x^2 + 9y^2 = 36$ , from  $(\sqrt{5}, 0)$  and  $(-\sqrt{5}, 0)$  is :  
 (A) 4 (B) 8  
 (C) 6 (D) 18
54. Two tangents from  $(5, 2)$  to the ellipse  $2x^2 + 7y^2 = 14$  are :  
 (A)  $x - y = 3; x + 9y = 13$  (B)  $x = y; x + 9y + 3 = 0$   
 (C)  $x + 9y = 0; x - y = 10$  (D)  $x - y = 3; x - 9y + 13 = 0$
55. Equation of chord of contact of tangents from the point  $(2, 4)$  to the ellipse  $2x^2 + 5y^2 = 20$  is :  
 (A)  $x + 5y = 5$  (B)  $-x + 5y - 5 = 0$   
 (C)  $x - 5y = 5$  (D)  $x = 5y$
56. Equation of the tangent and normal to the parabola  $x^2 + x - 2y + 2 = 0$  at  $(1, 2)$  :  
 (A)  $3x - 2y - 1 = 0$  (B)  $3x - 2y + 1 = 0$   
 $2x + 3y + 8 = 0$  (C)  $2x + 3y - 8 = 0$   
 (D)  $3x = 2y$  (E)  $2x + 3y = 0$
57. The point  $(2, 3, 4)$  is one end of the diameter of a sphere  $x^2 + y^2 + z^2 - 2x - 2y + 4z - 1 = 0$ ; the other end is :  
 (A)  $(0, -1, -8)$   
 (B)  $(1, 0, -8)$   
 (C)  $(8, 0, -1)$   
 (D)  $(0, 8, -1)$
58. Supplies are dropped from a helicopter and distance fallen in time  $t$  seconds is given by  $x = \frac{1}{2}gt^2$  where  $g = 9.8 \text{ m/sec}^2$ . Velocity and acceleration of the supplies after it has fallen for 2 seconds :  
 (A)  $9.6 \text{ m/sec}; 9.8 \text{ m/sec}^2$  (B)  $19.6; 16.9$   
 (C)  $19.6; 9.8$  (D)  $-18; 9$
59. Angle between the asymptotes of the hyperbola  $3x^2 - y^2 - 12x - 6y - 9 = 0$  is :  
 (A)  $\frac{\pi}{3}$  (B)  $2\pi$   
 (C)  $\frac{4\pi}{3}$  (D)  $\frac{2\pi}{3}$
60. Eccentricity of ellipse is less than :  
 (A) zero (B) one  
 (C) 1.5 (D) 2
61. If  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , represents ellipse  $b^2$  must be equal to :  
 (A)  $a^2(1 + e^2)$  (B)  $a^2/e^2$   
 (C)  $a^2(e^2 - 1)$  (D)  $a^2(1 - e^2)$

62. Which of the following is not a binary operation on  $\mathbb{R}$  ?
- (A)  $a * b = ab$  (B)  $a * b = a - b$   
 (C)  $a * b = \sqrt{ab}$  (D)  $a * b = \sqrt{a^2 + b^2}$
63. A monoid becomes a group if it also satisfies the :
- (A) Closure axiom (B) Associative axiom  
 (C) Identity axiom (D) Inverse axiom
64. Which of the following is not a group ?
- (A)  $(\mathbb{Z}_n, +_n)$  (B)  $(\mathbb{Z}, +)$   
 (C)  $(\mathbb{Z}, \cdot)$  (D)  $(\mathbb{R}, +)$
65. In the set of integers with operation  $*$  defined by  $a * b = (a + b - ab)$ , the value of  $3 * (4 * 5)$  is :
- (A) 25 (B) 15  
 (C) 10 (D) 5
66. The order of [7] in  $[\mathbb{Z}_9, +_9]$  is :
- (A) 9 (B) 6  
 (C) 3 (D) 1
67. In the multiplicative group of cube root of unity, the order of  $w^2$  is :
- (A) 4 (B) 3  
 (C) 2 (D) 1
68. The value of  $[3]_{+11} ([5]_{+11} [6])$  is :
- (A) [0] (B) [1]  
 (C) [2] (D) [3]
69. In the set of real numbers  $\mathbb{R}$ , an operation  $*$  is defined by  $a * b = \sqrt{a^2 + b^2}$ . Then the value of  $(3 * 4) * 5$  is :
- (A) 5 (B)  $5\sqrt{2}$   
 (C) 25 (D) 50
70. Which of the following is correct ?
- (A) An element of a group can have more than one inverse  
 (B) If every element of a group is its own inverse then the group is abelian  
 (C) The set of all  $2 \times 2$  real matrices forms a group under matrix multiplication  
 (D)  $(a * b)^{-1} = a^{-1} * b^{-1}$  for all  $a, b \in G$

71. The order of  $-i$ , in the multiplicative group of unity is :  
 (A) FOUR (B) THREE  
 (C) TWO (D) ONE
72. In the multiplicative group of  $n^{\text{th}}$  roots of unity, the inverse of  $w^k$  is; ( $k < n$ ):  
 (A)  $w^{1/k}$  (B)  $w^{-1}$   
 (C)  $w^{n-k}$  (D)  $w^{n/k}$
73. In the set of integers under the operation  $*$  defined by  $a * b = a + b - 1$ , the identity element is :  
 (A) zero (B) one  
 (C)  $a$  (D)  $b$
74. If  $S$  be a non-empty set and  $\circ$  be a binary operation on  $S$  defined by  $x \circ y = x$ ;  $x, y \in S$  then  $\circ$  is :  
 (A) Non-commutative but associative (B) Commutative  
 (C) Non-commutative (D) Commutative and associative
75. The Rolle's constant for the function  $y = x^2$  on  $[-2, 2]$  is :  
 (A)  $\frac{2\sqrt{3}}{3}$  (B) Zero  
 (C) Two (D)  $-2$
76. The 'C' of the Lagranges Mean value theorem for the function  $f(x) = x^2 + 2x - 1$ ;  $a = 0$ ;  $b = 1$  is :  
 (A)  $-1$  (B)  $1$   
 (C)  $0$  (D)  $\frac{1}{2}$
77. The value of 'C' in Rolle's theorem for the function  $f(x) = \cos\left(\frac{x}{2}\right)$  on  $[\pi, 3\pi]$  is :  
 (A)  $0$  (B)  $2\pi$   
 (C)  $\pi/2$  (D)  $3\pi/2$
78. The value of C of Lagranges Mean value theorem for  $f(x) = \sqrt{x}$  when  $a = 1$ , and  $b = 4$  is :  
 (A)  $\frac{9}{4}$  (B)  $\frac{3}{2}$   
 (C)  $\frac{1}{2}$  (D)  $\frac{1}{4}$

79.  $\lim_{x \rightarrow \infty} \left( \frac{x^2}{e^x} \right) = \dots\dots\dots ?$

- (A) 2 (B) 0  
(C)  $\infty$  (D) 1

80. The area bounded by the line  $y = x$ , then  $x$ -axis, the ordinates  $x = 1$ ,  $x = 2$  is :

- (A)  $3/2$  (B)  $5/2$   
(C)  $1/2$  (D)  $7/2$

81. If  $f(a) = 2$ ;  $f'(a) = 1$ ;  $g(a) = -1$ ;  $g'(a) = 2$  then the value of  $\lim_{x \rightarrow a} \left[ \frac{g(x) \cdot f(a) - g(a) \cdot f(x)}{x - a} \right]$  is equal to :

- (A) 5 (B)  $-5$   
(C) 3 (D)  $-3$

82. Which of the following function is increasing in  $(0, \infty)$  ?

- (A)  $e^x$  (B)  $\frac{1}{x}$   
(C)  $-x^2$  (D)  $x^{-2}$

83. The function  $f(x) = x^2 - 5x + 4$ , is increasing in :

- (A)  $(-\infty, 1)$  (B)  $(1, 4)$   
(C)  $(4, \infty)$  (D) everywhere

84. The function  $f(x) = x^2$ , is decreasing in :

- (A)  $(-\infty, \infty)$  (B)  $(-\infty, 0)$   
(C)  $(0, \infty)$  (D)  $(-2, \infty)$

85. The function  $y = \tan x - x$ , is :

- (A) an increasing function in  $\left( 0, \frac{\pi}{2} \right)$   
(B) a decreasing function in  $\left( 0, \frac{\pi}{2} \right)$   
(C) increasing in  $\left( 0, \frac{\pi}{4} \right)$  and decreasing in  $\left( \frac{\pi}{4}, \frac{\pi}{2} \right)$   
(D) decreasing in  $\left( 0, \frac{\pi}{4} \right)$  and increasing in  $\left( \frac{\pi}{4}, \frac{\pi}{2} \right)$

86. In a given semi-circle of diameter 4 cm a rectangle is to be inscribed. The maximum area of the rectangle is :
- (A) 2 (B) 4  
(C) 8 (D) 16
87. The least possible perimeter of a rectangle of  $100 \text{ m}^2$  is :
- (A) 10 (B) 20  
(C) 40 (D) 60
88. If  $f(x) = x^2 - 4x + 5$ , on  $[0, 3]$  then the absolute maximum value is :
- (A) 2 (B) 3  
(C) 4 (D) 5
89. The curve  $y = -e^{-x}$  is :
- (A) concave upward for  $x > 0$  (B) concave downward for  $x > 0$   
(C) everywhere concave upward (D) everywhere concave downward
90. Which of the following curves is concave down ?
- (A)  $y = -x^2$  (B)  $y = x^2$   
(C)  $y = e^x$  (D)  $y = x^2 + 2x - 3$
91. The point of inflexion of the curve  $y = x^4$ , is at :
- (A)  $x = 0$  (B)  $x = 3$   
(C)  $x = 12$  (D) nowhere
92. The curve  $y = ax^3 + bx^2 + cx + d$ , has a point of flexion at  $x = 1$ , then :
- (A)  $a + b = 0$  (B)  $a + 3b = 0$   
(C)  $3a + b = 0$  (D)  $3a + b = 1$
93. If  $u = x^y$ , then  $\frac{\partial u}{\partial x}$ , is equal to :
- (A)  $yx^{y-1}$  (B)  $u \log x$   
(C)  $u \log y$  (D)  $xy^{x-1}$
94. If  $u = \sin^{-1} \left( \frac{x^4 + y^4}{x^2 + y^2} \right)$  and  $f = \sin u$ , then  $f$  is a homogeneous function of degree :
- (A) 0 (B) 1  
(C) 2 (D) 4

95. If  $u = \frac{1}{\sqrt{x^2 + y^2}}$  then,  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ , is equal to :

(A)  $\frac{u}{2}$

(B)  $u$

(C)  $\frac{3u}{2}$

(D)  $-u$

96. The curve  $y^2(x - 2) = x^2(1 + x)$ , has :

(A) an asymptote parallel to x-axis

(B) an asymptote parallel to y-axis

(C) an asymptotes parallel to both axes

(D) no asymptotes

97. If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , then  $\frac{\partial r}{\partial x}$  is equal to :

(A)  $\sec \theta$

(B)  $\sin \theta$

(C)  $\cos \theta$

(D)  $\operatorname{cosec} \theta$

98. If  $u = \log \left( \frac{x^2 + y^2}{xy} \right)$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ , is :

(A) 0

(B)  $u$

(C)  $2u$

(D)  $\left( \frac{1}{u} \right)$

99. Air is being pumped into spherical balloon so that its volume increases at a rate of  $100 \text{ cm}^3/\text{sec}$ . How fast is the radius of the balloon is increasing when diameter is 500 m ?

(A)  $25\pi \text{ cm/s}$

(B)  $25 \text{ cm/s}$

(C)  $5\pi \text{ cm/sec}$

(D)  $\left( \frac{1}{25\pi} \right) \text{ cm/s}$

100. An asymptote to the curve  $y^2(a + 2x) = x^2(3a - x)$  is :

(A)  $x = 3a$

(B)  $x = -a/2$

(C)  $x = a/2$

(D)  $x = 0$

**ROUGH WORK**