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**COMBINED COMPETITIVE (PRELIMINARY) EXAMINATION, 2012**

Serial No.

**MATHEMATICS**

**Code No. 13**



*Time Allowed : Two Hours*

*Maximum Marks : 300*

**INSTRUCTIONS**

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
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*DO NOT* write *anything else* on the Test Booklet.
4. This Booklet contains 120 items (questions). Each item comprises *four* responses (answers). You will select *one* response which you want to mark on the Response Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.
5. In case you find any discrepancy in this test booklet in any question(s) or the Responses, a written representation explaining the details of such alleged discrepancy, be submitted within three days, indicating the Question No(s) and the Test Booklet Series, in which the discrepancy is alleged. Representation not received within time shall not be entertained at all.
6. You have to mark all your responses **ONLY** on the separate Response Sheet provided. *See directions in the Response Sheet.*
7. All items carry equal marks. Attempt **ALL** items. Your total marks will depend only on the number of correct responses marked by you in the Response Sheet.
8. Before you proceed to mark in the Response Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Response Sheet as per instructions sent to you with your Admit Card and Instructions.
9. While writing Centre, Subject and Roll No. on the top of the Response Sheet in appropriate boxes use **“ONLY BALL POINT PEN”**.
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## ROUGH WORK

1. In a group of 1000 people, there are 750 people who can speak Hindi and 400 who can speak English. Then number of people who can speak Hindi only is :
 

(A) 300	(B) 400
(C) 600	(D) 150
  
2. The composite mapping  $f \circ g$  of the mappings  $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = \sin x$  and  $g : \mathbb{R} \rightarrow \mathbb{R}, g(x) = x^2$  is :
 

(A) $\sin x + x^2$	(B) $(\sin x)^2$
(C) $\sin x^2$	(D) $\frac{\sin x}{x^2}$
  
3. Let  $\mathbb{R}$  be the set of real numbers. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = e^x$ , then  $f$  is :
 

(A) surjective but not injective	(B) injective but not surjective
(C) bijective	(D) neither surjective nor injective
  
4.  $n/m$  means that  $n$  is a factor of  $m$ , then the relation ' $\prime$ ' is :
 

(A) reflexive and symmetric
(B) transitive and symmetric
(C) reflexive, transitive and symmetric
(D) reflexive, transitive and not symmetric
  
5. If  $A$  and  $B$  are defined as :
 
$$A = \{(x, y) : y = e^x, x \in \mathbb{R}\}$$

$$B = \{(x, y) : y = x, x \in \mathbb{R}\}$$
 then :
 

(A) $B \subset A$	(B) $A \subset B$
(C) $A \cap B = \phi$	(D) $A \cup B = A$
  
6. The sets  $A$  and  $B$  have 3 and 6 elements respectively. What can be the minimum number of elements in  $A \cup B$  ?
 

(A) 18	(B) 9
(C) 6	(D) 3
  
7. If  $A, B$  and  $C$  are non empty subsets of a set then  $(A - B) \cup (B - A)$  equals :
 

(A) $(A \cap B) \cup (A \cup B)$	(B) $(A \cup B) - (A \cap B)$
(C) $A - (A \cap B)$	(D) $(A \cup B) - B$

8. The smallest positive integer  $n$  for which  $(1 + i)^{2n} = (1 - i)^{2n}$  is :
- (A) 4 (B) 8  
(C) 2 (D) 12
9. If  $iz^3 + z^2 - z + i = 0$ , where  $z$  is a complex number, then  $|z| =$
- (A) 1 (B)  $i$   
(C)  $-1$  (D)  $-i$
10. The equation  $|z + 1 - i| = |z + i - 1|$  represents :
- (A) a straight line (B) a circle  
(C) a parabola (D) a hyperbola
11. If  $\frac{1}{x} + x = 2 \cos \theta$ , then  $\frac{1}{x^n} + x^n$  is equal to :
- (A)  $2 \cos n\theta$  (B)  $2 \sin n\theta$   
(C)  $\cos n\theta$  (D)  $\sin n\theta$
12. If  $x^2 = e$  for all elements  $x$  of a group  $G$ , then :
- (A)  $G$  must be cyclic (B)  $G$  must be non-abelian  
(C)  $G$  must be abelian (D)  $G$  must be a finite group
13. Which of the following is not a group ?
- (A)  $(\mathbb{Z}_n, +_n)$  (B)  $(\mathbb{Z}, +)$   
(C)  $(\mathbb{Z}, \cdot)$  (D)  $(\mathbb{R}, +)$
14. The binary operation ' $\circ$ ' on  $\mathbb{R}$  defined by  $x \circ y = x^y + y^x$  is :
- (A) Commutative and associative (B) Commutative but not associative  
(C) Associative but not commutative (D) Neither commutative nor associative
15. The characteristic of the ring  $2\mathbb{Z}$  is :
- (A) 2 (B) 3  
(C) 1 (D) 0
16. If  $A$  and  $B$  are matrices such that  $AB = A$ ,  $BA = B$  then  $B^2$  is equal to :
- (A)  $B$  (B)  $A$   
(C)  $I$  (D)  $O$

17. If the product matrix  $AB$  is zero, then :
- (A)  $A = 0$  or  $B = 0$   
 (B) It is not necessary that either  $A = 0$  or  $B = 0$   
 (C)  $A = 0$  and  $B = 0$   
 (D) All the above statements are false
18. If  $A$  and  $B$  are two non singular matrices of order  $n$  then :
- (A)  $AB$  is non-singular (B)  $AB$  is singular  
 (C)  $(AB)^{-1} = A^{-1} B^{-1}$  (D)  $(AB)^{-1}$  does not exist
19. If  $A$  and  $B$  are  $2 \times 2$  matrices then  $\det(A + B) = 0$  implies :
- (A)  $\det A = 0$  and  $\det B = 0$  (B)  $\det A + \det B = 0$   
 (C)  $\det A = 0$  or  $\det B = 0$  (D) None of these

20. If \_\_\_\_\_, then the value of  $X^n$  is :

(A)  $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$

(B)  $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$

(C)  $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$   $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$

(D) None of these

21. The system of equations :

$$x + y + z = 2$$

$$2x + y - z = 3$$

$$3x + 2y + kz = 4$$

has a unique solution, if :

(A)  $k \neq 0$

(B)  $-1 < k < 1$

(C)  $-2 < k < 2$

(D)  $k = 0$

22. The system :

$$x + 2y + 3z = 1$$

$$x - y + 4z = 0$$

$$2x + y + 7z = 1$$

have :

(A) only two solutions

(B) only one solution

(C) no solution

(D) infinitely many solutions

23. If  $AB = 0$  for the matrices :

$$A = \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix}, \quad B = \begin{bmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}$$

then  $\theta - \phi$  is :

- (A) an odd multiple of  $\frac{\pi}{2}$                       (B) an odd multiple of  $\pi$   
 (C) an even multiple of                      (D) 0

24. The rank of the matrix                      is :

- (A) 3    (B) 2  
 (C) 1    (D) -2

25. The value of the determinant of  $n^{\text{th}}$  order  $\begin{vmatrix} x & 1 & 1 & \dots & \dots & \dots \\ 1 & x & 1 & \dots & \dots & \dots \\ 1 & 1 & x & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{vmatrix}$  is :

- (A)  $(x - 1)^{n-1} (x + n - 1)$                       (B)  $(x - 1)^n (x + n - 1)$   
 (C)  $(1 - x)^{n-1} (x + n - 1)$                       (D)  $(1 - x)^n (x + n - 1)$

26. If  $1, w, w^2$  are the cube roots of unity, then  $D = \begin{vmatrix} 1 & w^n & w^{2n} \\ w^{2n} & 1 & w^n \\ w^n & w^{2n} & 1 \end{vmatrix}$  has the value :

- (A) 0    (B) w  
 (C)  $w^2$     (D) 1

27. If  $\Delta = \begin{vmatrix} 1 & 3 \cos \theta & 1 \\ \sin \theta & 1 & 3 \cos \theta \\ 1 & \sin \theta & 1 \end{vmatrix}$  then maximum value of  $\Delta$  is :

- (A) 1    (B) 9  
 (C) 16    (D) None of these

28. If  $d$  is the determinant of a square matrix  $A$  of order  $n$ , then the determinant of its adjoint is :

- (A)  $d^n$  (B)  $d^{n-1}$   
 (C)  $d^{n+1}$  (D)  $d$

29. If  $A$  is a square matrix of order 3 and  $A = 5B$  then  $|A| =$

- (A)  $5|B|$  (B)  $25|B|$   
 (C)  $125|B|$  (D) None of these

30. If  $f(x) = x^2 - 4x + 1$  and  $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$  then  $f(A)$  is :

- (A) null matrix (B) identity matrix  
 (C)  $\begin{bmatrix} 4 & 6 \\ -2 & 4 \end{bmatrix}$  (D) none of these

31. The domain of the function  $f(x) = \sqrt{x-1} + \sqrt{5-x}$  is :

- (A)  $[1, \infty)$  (B)  $(-\infty, 5)$   
 (C)  $(1, 5)$  (D)  $[1, 5]$

32. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  satisfies the relation  $f(x+y) = f(x) + f(y)$ , for all  $x, y \in \mathbb{R}$  and  $f(1) = 7$ , then  $\lim_{x \rightarrow 0} \frac{7n(n+1)}{2x} \sum_{r=1}^n f(r)$  is :

- (A)  $\frac{7n}{2}$  (B)  $\frac{7(n+1)}{2}$   
 (C)  $7n(n+1)$  (D)

33. The cardinality of the set  $A = \{\phi\}$  is :

- (A) 0 (B) 1  
 (C) -1 (D) 2

34. is :

- (A)  $(n-1)!$  (B)  $n!$   
 (C)  $n$  (D)  $-n$

35. If  $f(x) = \begin{cases} x, & \text{when } x \text{ is rational} \\ 1-x, & \text{when } x \text{ is irrational} \end{cases}$
- then :
- (A)  $f$  is continuous for all reals  $x$   
 (B)  $f$  is discontinuous for all reals  $x$   
 (C)  $f$  is continuous only at  $x = \frac{1}{2}$   
 (D)  $f$  is discontinuous only at  $x = \frac{1}{2}$
36. Let  $f = \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by  $f(x) = \max \{x, x^3\}$ . The set of all points where  $f(x)$  is not differentiable is :
- (A)  $\{-1, 1\}$  (B)  $\{-1, 0\}$   
 (C)  $\{0, 1\}$  (D)  $\{-1, 0, 1\}$
37. If  $y = \sin(m \sin^{-1} x)$ , then  $(1 - x^2) y'' - xy'$  is equal to :
- (A)  $m^2 y$  (B)  $my$   
 (C)  $-m^2 y$  (D) None of these
38. If  $y = \cos x \cos 2x \cos 4x \cos 8x$ , then  $f'\left(\frac{\pi}{4}\right)$  is :
- (A)  $-1$  (B)  $2$   
 (C)  $\sqrt{2}$  (D)  $\frac{1}{2}$
39. The greatest value of  $f(x) = x^5 - 5x^4 + 5x^3 + 12$  on  $[0, 1]$  is :
- (A)  $13$  (B)  $1$   
 (C)  $0$  (D)  $-13$
40. The set of values of  $x$  for which  $\log(1+x) < x$ , is :
- (A)  $x < 0$  (B)  $x > 0$   
 (C)  $0 < x < 1$  (D)  $x < 1$
41. If  $p(x) = a_0 + a_1 x^2 + a_2 x^4 + a_3 x^6 + \dots + a_n x^{2n}$  be a polynomial in  $x \in \mathbb{R}$  with  $0 < a_1 < a_2 < \dots < a_n$ , then  $p(x)$  has :
- (A) no point of minimum (B) only one point of minimum  
 (C) only two points of minimum (D) none of these



42. The function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = (x + 2)e^{-x}$  is :
- (A) decreasing for all  $x$   
 (B) decreasing in  $(-\infty, -1)$  and increasing in  $(-1, \infty)$   
 (C) increasing for all  $x$   
 (D) decreasing in  $(-1, \infty)$  and increasing in  $(-\infty, -1)$
43. The lines  $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$  and  $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$  are coplanar if :
- (A)  $k = 0$  or  $-1$  (B)  $k = 1$  or  $-1$   
 (C)  $k = 0$  or  $-3$  (D)  $k = 3$  or  $-3$
44. The equation of the normal to the curve  $x^2 = 4y$  passing through the point  $(1, 2)$  is :
- (A)  $x + y + 3 = 0$  (B)  $x - y - 3 = 0$   
 (C)  $2x + y = 4$  (D)  $2x - y = 1$
45. Area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is :
- (A)  $\pi ab$  (B)  $\frac{\pi ab}{2}$   
 (C) (D)  $\frac{\pi ab}{3}$
46. If  $f(x)$  is an odd function, then  $\int_a^b f(t) dt$  is :
- (A) an odd function (B) even function  
 (C) neither even nor odd (D) none of these
47. Find the area of the segment of the parabola  $y = x^2 - 5x + 15$  cut off by a straight line  $y = 3x + 3$  :
- (A)  $\frac{32}{3}$  (B) 0  
 (C) 1 (D) None

48. If  $f(x) = \begin{cases} e^{\cos x} \cdot \sin x, & |x| \leq 2 \\ 2 & , \text{ otherwise} \end{cases}$  then  $\int_{-2}^3 f(x) dx$  is equal to :

- (A) 0 (B) 1  
(C) 2 (D) 3

49. Let  $g(x) = \int_0^x f(t) dt$ , where  $f(t) = \begin{cases} \left[ \frac{1}{2}, 1 \right], & \forall t \in [0, 1] \\ \left[ 0, \frac{1}{2} \right], & \forall t \in (1, 2] \end{cases}$

then :

- (A)  $g(2) \in \left[ \frac{-3}{2}, \frac{1}{2} \right)$  (B)  $g(2) \in \left[ \frac{1}{2}, \frac{3}{2} \right]$   
(C)  $g(2) \in \left[ \frac{3}{2}, \frac{5}{2} \right)$  (D)  $g(2) \in (2, 4)$

50. Let .

Then real roots of the equation  $x^2 - f'(x) = 0$  are :

- (A)  $\pm 1$  (B)  $\pm \frac{1}{\sqrt{2}}$   
(C)  $\pm \frac{1}{2}$  (D) 0 and 1

51. Let  $T > 0$  be a fixed number. Suppose  $f$  is a continuous function, such that for all  $x \in \mathbb{R}$ ,

$f(x + T) = f(x)$ . If  $I = \int_0^T f(x) dx$  then the value of  $\int_3^{3+3T} f(2x) dx$  is :

- (A)  $\frac{3}{2}I$  (B)  $2I$   
(C)  $3I$  (D)  $6I$

52. The area of region bounded by  $y = |x - 1|$  and  $y = 1$  is :

- (A) 2 (B) 1  
(C)  $\frac{1}{2}$  (D)  $\frac{1}{3}$

53. The value of  $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$  is equal to :

- (A)  $\frac{\pi}{4}$  (B)   
 (C) (D)

54. If  $I_n = \int_0^{\pi/2} \sin^n x dx$ , then  $I_n + I_{n-2} =$

- (A) 1 (B)  $n - 1$    
 (C)  $\frac{1}{n-1}$  (D)  $\frac{1}{n(n-1)}$

55.  $\int_0^{2\pi} \frac{dx}{1 + e^{\sin x}}$  is equal to :

- (A)  $\pi$  (B)  $2\pi$    
 (C)  $\frac{\pi}{2}$  (D)  $-\pi$

56.  $\int_{-1}^1 \frac{dx}{\sqrt{1-x^2}}$  is :

- (A) 1 (B) 0   
 (C) -1 (D)  $\frac{1}{2}$

57.  $\int_0^1 x e^{2x} dx$  is :

- (A)  $\frac{1}{4}(1+e^2)$  (B)  $\frac{1}{4}(1-e^2)$    
 (C) 0 (D)  $\frac{1}{4}(1+e^{-2})$

58.  $\int_0^{\pi/2} \cos^5\left(\frac{x}{2}\right) \cdot \sin x \, dx$  is equal to :

(A)  $\frac{2}{7}\left(1 - \frac{1}{8\sqrt{2}}\right)$

(B)  $-\frac{4}{7}\left(1 - \frac{1}{8\sqrt{2}}\right)$

(C)  $\frac{4}{7}\left(1 - \frac{1}{8\sqrt{2}}\right)$

(D)  $-\frac{2}{7}\left(1 - \frac{1}{8\sqrt{2}}\right)$

59.  $\int_0^{\pi/2} \sin 2x \log \cot x \, dx$  is :

(A)  $\frac{1}{2}$

(B) 1

(C) 0

(D)  $-\frac{1}{2}$

60. The area bounded by the curves  $y^2 = 8x$  and  $x^2 = 8y$  is :

(A)  $\frac{32}{7}$

(B)  $\frac{24}{7}$

(C)  $\frac{72}{7}$

(D)  $\frac{64}{3}$

61. Let  $a_n$  be the  $n^{\text{th}}$  term of the G.P. of positive numbers. Let  $\sum_{n=1}^{100} a_{2n} = \alpha$  and  $\sum_{n=1}^{100} a_{2n-1} = \beta$

such that  $\alpha \neq \beta$ , then the common ratio is :

(A)  $\frac{\alpha}{\beta}$

(B)

(C)

(D)  $\sqrt{\frac{\beta}{\alpha}}$

62. The sum of the first  $n$  terms of the series  $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$  is :

(A)  $2^n - 1$

(B)  $1 - 2^{-n}$

(C)  $2^{-n} - n + 1$

(D)  $2^{-n} + n - 1$

63. If  $\frac{1}{a^x} = \frac{1}{b^y} = \frac{1}{c^z}$  and a, b, c are in G.P., then x, y, z are in :

- (A) A.P. (B) G.P.  
 (C) H.P. (D) None of these

64. The next term of the sequence 1, 5, 14, 30, 55, ..... is :

- (A) 91 (B) 85  
 (C) 90 (D) 95

65. The value of :

$$1 + (1 + 2) + (1 + 2 + 3) + \dots + (1 + 2 + 3 + \dots + n)$$

is :

- (A)  $n(n + 1)$  (B)  $\frac{n(n + 1)(n + 2)}{6}$   
 (C) 1 (D)

66. If  $S_n = \sum_{r=1}^n \frac{2r+1}{r^4}$  then  $S_{20}$  is equal to :

- (A)  $\frac{220}{221}$  (B)  $\frac{420}{441}$   
 (C)  $\frac{439}{221}$  (D)  $\frac{440}{441}$

67. The infinite series  $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$  :

- (A) Converges for all  $x \in \mathbb{R}$  (B) Converges for all  $x \in [-1, 1)$   
 (C) Diverges for all  $x \in [-1, 1)$  (D) Converges for all  $x \in [-1, 1]$

68. The series  $1 + x + x^2 + x^3 + \dots$  is the expansion of the function :

- (A)  $\frac{1}{1+x}$  (B)  $\frac{1}{1-x}$   
 (C)  $\frac{1}{x-1}$  (D)  $\frac{1}{-1-x}$

69. The series  $\sum_{n=2}^{\infty} \frac{1}{n \log n}$  is :
- (A) convergent (B) divergent  
(C) absolutely convergent (D) oscillatory
70. The series  $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots$  is :
- (A) convergent (B) divergent  
(C) absolutely convergent (D) oscillatory
71. The degree of the differential equation  $x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3$  is :
- (A) Three (B) One  
(C) Not defined (D) None of these
72. The differential equation representing the family of curves  $y^2 = 2c(x + \sqrt{c})$ , where  $c$  is a positive parameter is of :
- (A) order 1, degree 3 (B) order 2, degree 3  
(C) order 3, degree 1 (D) order 1, degree 1
73. Solution of the equation  $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$  is :
- (A)  $e^y = e^x - 1 + c e^{-e^x}$  (B)  $e^y = e^x - 1 + c e^{e^x}$   
(C)  $e^x = e^y - 1 + c e^{-e^y}$  (D)  $e^x = e^y - 1 + c e^{e^y}$
74. The solution of  $\frac{dy}{dx} = \frac{ax + g}{by + f}$  represents a circle when :
- (A)  $a = b$  (B)  $a = -b$   
(C)  $a = -2b$  (D)  $a = 2b$
75. If  $2f(x) = f'(x)$  and  $f(0) = 3$ , then  $f(2)$  equals :
- (A)  $4 e^3$  (B)  $3 e^4$   
(C)  $2 e^3$  (D)  $3 e^2$

76. Integrating factor of the differential equation  $\cos x \frac{dy}{dx} + y \sin x = 1$  is :

- (A)  $\sin x$  (B)  $\sec^2 x$   
(C)  $\tan x$  (D)  $\cos x$

77. The differential equation  $(3a^2x^2 + by \cos x) dx + (2 \sin x - 4ay^3) dy = 0$  is exact for :

- (A) any value of  $a$  and  $b \neq 2$  (B) any value of  $a$  and  $b = 2$   
(C) any value of  $b$  and  $a = 2$  (D) any values of  $a$  and  $b$

78. General solution of  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  is of the form :

- (A)  $u = f(x + iy) + g(x - iy)$   
(B)  $u = f(x - iy) + g(x + iy)$   
(C)  $u = f(x + iy) - g(x - iy)$   
(D)  $u = f(x - iy) - g(x + iy)$

79. Orthogonal trajectories of  $xy = c^2$  are :

- (A)  $x^2 = y + \text{constant}$  (B)  $x^2 + y^2 = \text{constant}$   
(C)  $x^2 - y^2 = \text{constant}$  (D)  $x - y = \text{constant}$

80. If  $y = A \sin x + B \cos x$ , then the value of  $\frac{d^2 y}{dx^2} + y$  is :

- (A)  $\sin x + \cos x$  (B) 2  
(C) 1 (D) 0

81. If  $x = a \sec \theta$ ,  $y = b \tan \theta$  then  $\frac{dy}{dx}$  is :

- (A)  $\frac{b}{a} \operatorname{cosec} \theta$  (B)  $\frac{a}{b} \tan \theta$   
(C)  $\frac{a}{b}$  (D)  $\frac{a}{b} \operatorname{cosec} \theta$

82. The differential equation of all lines in a plane which are at a constant distance  $p$  from origin is :

(A)  $(p^2 - x^2) \left( \frac{dy}{dx} \right)^2 - 2xy \frac{dy}{dx} + p^2 - y^2 = 0$

(B)  $(p^2 - x^2) \frac{dy}{dx} + y = 0$

(C)  $\left( \frac{dy}{dx} \right)^2 - 2xy \frac{dy}{dx} + y^2 = 0$

(D) None

83. The set  $G = \{1, -1, i, -i\}$  is a group with respect to :

(A) addition

(B) subtraction

(C) division

(D) multiplication

84. Which one of the following is true ?

(A) Every field is an integral domain

(B) Every integral domain is a field

(C)  $\mathbb{Z}$  is a subfield of  $\mathbb{Q}$

(D) All of these

85. The symmetric group  $S_3$  has :

(A) 4 elements

(B) 6 elements

(C) 3 elements

(D) 8 elements

86. The order and degree of the differential equation  $\frac{dy}{dx} + \int y dx = x$  is :

(A) order 1, degree 1

(B) order 2, degree 1

(C) order 2, degree 2

(D) order 1, degree 2

87. The differential equation  $(x^3 + 3y^2x)dx + (y^3 + 3x^2y)dy = 0$  is :

(A) not exact

(B) not homogeneous

(C) both exact and homogeneous

(D) neither exact nor homogeneous

88. The integrating factor of the equation  $y \log y dx + (x - \log y)dy = 0$  is :

(A)  $\frac{1}{y^2}$

(B)  $\frac{1}{x^2}$

(C)  $\frac{1}{xy}$

(D)  $\frac{1}{y}$



89. The equation of the curve of degree 2 which passes through (0, 3), (1, 6), (2, 11) and (3, 18) is :
- (A)  $x^2 + 5x + 3$  (B)  $x^2 + 2x + 3$   
 (C)  $x^2 + x + 3$  (D)  $x^2 + 3x + 3$
90. The points  $\left(0, \frac{8}{3}\right)$ , (1, 3) and (82, 30) are vertices of :
- (A) an obtuse angled triangle (B) an acute angled triangle  
 (C) a right angled triangle (D) none of these
91. The gradient of one of the lines  $x^2 + hxy + 2y^2 = 0$  is twice that of the other, then  $h =$
- (A)  $\pm 3$  (B)  $\pm \frac{3}{2}$   
 (C)  $\pm 2$  (D)  $\pm 1$
92. The maximum value of  $z = 0.5y - 0.1x$  subject to the constraints  $2x + 5y \leq 80$ ,  $x + y \leq 20$ ,  $x, y \geq 0$  is :
- (A) 2 (B) 6  
 (C) 8 (D) 10
93. The angle between the lines represented by the equation  $\lambda x^2 + (1 - \lambda)xy - \lambda y^2 = 0$  is :
- (A)  $30^\circ$  (B)  $60^\circ$   
 (C)  $90^\circ$  (D)  $45^\circ$
94. The circles  $x^2 + y^2 - 10x + 16 = 0$  and  $x^2 + y^2 = r^2$  intersect each other in two distinct points if :
- (A)  $r < 2$  (B)  $r > 8$   
 (C)  $2 < r < 8$  (D)  $2 \leq r \leq 8$
95. If the equation  $kx^2 + 2y^2 - 5xy + 5x - 7y + 3 = 0$  represents two straight lines, then the value of  $k$  will be :
- (A) 0 (B) 2  
 (C) 4 (D) -6
96. The equations of tangents to the circle  $x^2 + y^2 = 25$  which are inclined at angle of  $30^\circ$  to the X-axis are :
- (A)  $y = x\sqrt{3} \pm 5$  (B)  $\sqrt{3}y = x \pm 10$   
 (C)  $\pm\sqrt{3}y = x + 10$  (D)  $y = \sqrt{3}x \pm 10$

97. Two circles  $x^2 + y^2 - 2x - 4y = 0$  and  $x^2 + y^2 - 8y - 4 = 0$  :
- (A) touch externally (B) touch internally  
(C) intersect (D) do not touch
98. The curve described parametrically by  $x = t^2 + t + 1$ ,  $y = t^2 - t + 1$  represents :
- (A) a pair of straight lines (B) an ellipse  
(C) a parabola (D) a hyperbola
99. If the normals at the point  $(at^2, 2at)$  meets the parabola again at  $(at_1^2, 2at_1)$  then :
- (A)  $t_1 + t = -\frac{2}{t}$  (B)  $t_1 = t + \frac{2}{t}$   
(C)  $t_1 = -\frac{2}{t}$  (D)  $t - t_1 = 2$
100. Cosine of the angle which the vector  $\vec{a} = 3\vec{i} - 6\vec{j} + 2\vec{k}$  makes with X axis is :
- (A)  $-\frac{6}{7}$  (B)  $\frac{3}{7}$   
(C)  $\frac{2}{7}$  (D)  $\frac{1}{7}$
101. The direction ratios of the line joining the points  $(2, 2, 1)$  and  $(3, -1, 3)$  are :
- (A)  $2, -1, 4$  (B)  $1, 1, 0$   
(C)  $0, 1, 0$  (D)  $1, -3, 2$
102. The perpendicular distance of the point  $(2, -3, 1)$  from the plane  $x - 2y - 3z - 10 = 0$  is :
- (A)  $\frac{4}{\sqrt{13}}$  (B)  $\frac{6}{\sqrt{2}}$   
(C)  $\frac{5}{\sqrt{14}}$  (D)  $2$
103. The equation of the plane passing through the point  $(2, 3, 4)$  and parallel to the plane  $x + 2y + 4z = 5$  is :
- (A)  $x + 2y + 4z = 10$  (B)  $x + 2y + 4z = 3$   
(C)  $x + y + 2z = 2$  (D)  $x + 2y + 4z = 24$

104. The equation of the cylinder whose generators are parallel to z-axis and intersects the curve  $ax^2 + by^2 = 2z$ ,  $lx + my + nz = p$  is :
- (A)  $2lx + 2my + n(ax^2 + by^2) = 0$       (B)  $2lx + 2my + n(ax^2 + by^2) = 2p$   
 (C)  $lx + my + nz + ax^2 + by^2 = 0$       (D) None of these
105. If two forces acting at right angles have their resultant  $\sqrt{10}$  kg. wt. and when they act at an angle of  $60^\circ$ , the resultant is  $\sqrt{13}$  kg. wt., then the forces are :
- (A) 3 kg. wt., 1 kg. wt.      (B) 2 kg. wt., 2 kg. wt.  
 (C) 4 kg. wt., 1 kg. wt.      (D) None of these
106. Three like parallel forces P, Q, R act at the corner points of a triangle ABC. Their resultant passes through the circumcenter if :
- (A)  $\frac{P}{2} = \frac{Q}{3} = \frac{R}{4}$       (B)  $P = Q = R$   
 (C)  $P + Q + R = 0$       (D)  $P = \frac{1}{2}(Q + R)$
107. A force  $3\bar{i} + 4\bar{j} + \bar{k}$  displaces a particle with displacement  $p\bar{i} + 3\bar{j} + \bar{k}$ . If the work done is 10 units, the value of p is :
- (A) -1      (B) 1  
 (C) 2      (D) 3
108. The maximum horizontal range of a projectile when the velocity of projection is 28 m/sec is :
- (A) 40 m      (B) 80 m  
 (C) 100 m      (D) 60 m
109. If T is the time of flight, R the horizontal range and  $\alpha$  the angle of projection of a particle, then  $\tan \alpha =$
- (A)  $\frac{gT}{2R}$       (B)  $\frac{gT^2}{R}$   
 (C)  $\frac{gT^2}{2R}$       (D)  $\frac{T^2}{2Rg}$

110. If the resultant of two forces of magnitude P and 2P is perpendicular to P, then the angle between the forces is :

- (A)  $\frac{2\pi}{3}$  (B)  $\frac{\pi}{3}$   
 (C)  $\frac{\pi}{6}$  (D)  $\frac{\pi}{4}$

111. A 1000 kg car goes from 10 m/s to 20 m/s in 5 sec. The force acting on it is :

- (A) 2000 N (B) 1000 N  
 (C) 1500 N (D) 1800 N

112. The maximum velocity of a particle executing simple harmonic motion with an amplitude 7 mm is 4.4 m/sec. The period of oscillation is :

- (A) 100 s (B) 0.01 s  
 (C) 10 s (D) 0.1 s

113. A function f(x) is continuous in [0, 1] and differentiable in (0, 1). Then there is a point c in (0, 1) such that f'(c) =

- (A) f(1) – f(0) (B) f(0) + f(1)  
 (C)  $\frac{f(1) - f(0)}{1}$  (D)  $\frac{f(1) - f(0)}{3}$

114. The power series expansion of sin x is :

- (A)  $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$  (B)  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$   
 (C)  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$  (D) None of these

115. If the vector  $\bar{i} - \bar{j} + \bar{k}$ ,  $3\bar{i} - 4\bar{j} + 5\bar{k}$  and  $\bar{i} + 2\bar{j} + t\bar{k}$  are coplanar then the value of t is :

- (A) 5 (B) -5  
 (C) 2 (D) 4

116. If l, m, n are the direction cosines of a line then :

- (A)  $l^2 + m^2 + n^2 = 0$  (B)  $l^2 + m^2 - n^2 = 1$   
 (C)  $l^2 + m^2 + n^2 = 1$  (D)  $l^2 - m^2 - n^2 = 1$

117. The distance of the point (1, 1, 4) from the plane  $3x - 6y + 2z + 11 = 0$  is :

(A) 3

(B) 4

(C)  $\frac{7}{16}$

(D)  $\frac{16}{7}$

118. The equation of the sphere with center (0, 0, 0) and radius 1 is :

(A)  $x^2 + y^2 - z^2 = 1$

(B)  $x^2 - y^2 + z^2 = 1$

(C)  $x^3 + y^3 + z^3 = 1$

(D)  $x^2 + y^2 + z^2 = 1$

119. If  $\alpha$  and  $\beta$  are roots of a quadratic equation  $ax^2 + bx + c = 0$ , then  $\alpha + \beta$  equals :

(A)  $\frac{a}{b}$

(B)  $\frac{c}{a}$

(C)  $-\frac{b}{a}$

(D)  $-\frac{a}{b}$

120. The volume of a right circular cone with radius r and height h is given by :

(A)  $\pi r^2 h$

(B)  $\frac{1}{2} \pi r^2 h$

(C)  $\frac{3}{4} \pi r h$

(D)  $\frac{1}{3} \pi r h$

## ROUGH WORK

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