

## **SYLLABUS FOR LECTURER 10+2 MATHEMATICS**

### **REAL ANALYSIS**

Resume of sequences, series and Riemann integration, continuity, uniform continuity. Fundamental theorem of integral calculus, classes of R-integrable functions. Functions of bounded variation. Riemann-Stieltjes integration. Cauchy's general principle of uniform convergence, uniform convergence and integration, uniform convergence and differentiation, Weierstrass theorem.

### **COMPLEX ANALYSIS**

Complex numbers and functions. Cauchy integral formula, Liouville's theorem, Taylor's and Laurent's theorems, classification of singularities. Removable singularities, Riemann's theorem, essential singularity, Weierstrass theorem on essential singularity. Calculus of residues, Cauchy's residue theorem, Integration by the method of residues, evaluation of  $\int_{-\infty}^{\infty} f(x) dx$  by residue calculus. The argument Principle of Maximum Modulus theorem, for bounded regions.

### **GROUPS**

Review of the product of two subgroups. Structure theorem for cyclic groups, automorphisms, inner automorphisms. Cauchy's and Sylow's theorem for Abelian groups. Cayley's theorem. Simplicity of the alternating groups. Sylow's theorem and Cauchy's theorem. Finite Abelian groups, Fundamental theorem on finite Abelian groups. Composition series. The Jordan-Hölder theorem for finite groups.

### **RINGS**

Definition and examples of rings. Integral domains and fields Homomorphisms, Principal ideals, Prime ideals and maximal ideals, Fields of quotients of an integral domains, Polynomial rings, Eisenstein's criterion.

### **FIELDS**

Prime fields and their structure. Extensions of fields. Algebraic numbers and algebraic extensions of a field. Normal extensions and fundamental theorem of Galois theory.

### **ADVANCED CALCULUS**

Fourier series: Expansion of function  $f(x)$  in the interval  $(- \pi, \pi)$ . Fourier sine series and Fourier Cosine series of  $f(x)$  in  $(- \pi, \pi)$ . Fourier series of sine and cosine in any interval  $(-c, c)$

### **DIFFERENTIAL EQUATIONS**

An initial value problem, singular solutions, P-discriminant, C-discriminant. Equations of the second degree with variable coefficients. Total differential equation.

$Pdx + Qdy + Rdz = 0$ . Necessary and sufficient conditions that such a differential equation may be integrable.

Partial differential equation of the first order, Lagrange's linear equation  $Pp + Qq + R = 0$ . Charpit's method.

### **DIFFERENTIAL GEOMETRY:**

Curves with torsion, curvature, Frenet formulae, spherical curvature, spherical indicatrices, Involutives and evolutes. Bertrand curves. Envelopes of one and two parameter family of surfaces, Developable surfaces, Developable associated with a curve. Curvilinear coordinates. Fundamental magnitudes of first and second order, the two fundamental forms; curvature of normal section, Meunier's theorem, Euler's theorem, Dupin's indicatrix, Rodrigue's formulae. Conjugate systems. Asymptotic lines, Isometric lines, null lines. The Gauss characteristic equation, Minardi-Codazzi Relations. Geodesic curves in relation to Geodesics. Bonnet's theorem. Geodesic curvature.

### **TOPOLOGY**

Metric spaces: Definition and examples, open sets, completeness, convergence, continuous mapping, completion of a metric space, Cantor's intersection theorem. Banach's contraction Principle.

## **TOPOLOGICAL SPACES**

Definition and examples, Elementary properties, Kuratowski's axioms, continuous mappings and their characterisation. Bases and subbases, concept of first countability, second countability, separability, Tychonoff's theorem, compactness, Lebesgue's covering lemma, continuous maps on compact spaces, Connectedness, local connectedness, their relationship. Urysohn's lemma, Urysohn's Metrization theorem. Separation axioms, one point compactification.

## **BANACH SPACES**

Definition and examples, Quotient spaces, Dual of a normed linear space. Duals of  $L^p$ ,  $C(X)$ ,  $\ell^p$  ( $1 \leq p < \infty$ ). Hahn Banach Theorem.

## **HILBERT SPACES**

Definition and examples, Cauchy-Schwarz inequality Bessel's inequality, orthonormal systems. Riesz representation theorem, inner product spaces, Adjoint of a Hilbert space, operators, Normal operators.

## **MEASURE THEORY**

Lebesgue outer measure, Lebesgue measurable set, Measurable functions, algebra of measurable functions. Borel Measurability. Convergence in measure, almost uniform convergence and Egorov's theorem. Lebesgue integration; Lebesgue integral of non-negative measurable function. Fatou's lemma, Lebesgue's Monotone convergence, Lebesgue's Dominated convergence theorem. Riemann and Lebesgue integrals. R-integrability of bounded functions. Lebesgue integrability of bounded functions. Lebesgue integrability  $L^p$  - spaces. Fubini's theorem.

## **PRIME NUMBERS**

Diophantine equations, solvability of linear Diophantine equations. Congruences, Fermat's theorem, Wilson's theorem, Primitive roots.

## **COMPLEX ANALYSIS**

The maximum modulus theorem. Schwarz lemma Hadamard's three circle theorem, Theorem of Borel and Carathéodory, Theorem of Phragmén-Lindelöf. Power series, Hadamard formula for the radius of convergence, a power series represents an analytic function within the circle of convergence. Hadamard's theorem i.e. If  $f(z) = \sum a_n z^n$  has radius of convergence equal to 1, and  $a_n$  is real with  $\sum a_n z^n$  properly divergent; then  $z = 1$  is a regularity. Rouché's theorem, the fundamental theorem of algebra. Morera's theorem is Poisson's integral formula for a circled and half plane, Poisson-Jensen formula.

## **ENTIRE FUNCTIONS**

Factorization of integral functions. The theorem of Weierstrass. The order of an entire function. Hadamard's factorization theorem, the order of a canonical product is equal to the exponent of convergence of its zeros. Order of the derived function. Picard's theorem.

## **UNIFORM SPACES**

Definition and examples, Uniform Topology; Uniformity and metrizability, complete regularity of uniform spaces, compactness in uniform spaces, uniform continuity, Homotopy theory; Brouwer's fixed point theorem.

## **BANACH ALGEBRA**

Preliminaries on Banach Algebras, Invertible elements, the spectrum, spectral radius and a formula for the spectral radius, Gelfand-Mazur theorem, Gelfand mapping, Maximal ideal space and its characterisation, continuity of multiplicative functions on Banach Algebra. Gelfand-Naimark theorem. Ideals in  $C(X)$  and application to Stone-Čech compactification. Spectral theorem for normal operators.

## **MODULES**

Definitions, fundamental concepts, chain conditions, Noetherian rings, Prime and primary ideals, Sequence theorems.

## **LATTICES**

Partially ordered sets, Lattices, modular lattices, complemented modular lattice.

**RING THEORY**

Rings, Hilbert basis theorem for Noetherian rings, Matrix rings and their ideals. Direct sums of rings, Prime radical of a ring.

Random Variables, Mathematical Expectation, cheboyshev's Inequality. Conditional probability, Baye's Theorem; The correlation, coefficient; Independence.

Binomial, Gamma and Chi-square distributions. Bivariate Normal Distributions., The t and F distributions. The moment Generating Function Technique.

The central Limit Theorem, Point Estimation, The Rao-Blockwell Theorem.

Further topics in point estimation Maximum likelihood Estimation statistical Hypothesis; Examples and definition, Uniformly most powerful Tests; Likelihood Ratio Tests.

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