Roll No. .... **Total No. of Pages : 5**

1(CCE.M)3

Mathematics–I

(15)

Time : Three Hours] [Maximum Marks : 300

INSTRUCTIONS

(i) Answers must be written in English.

(ii) The number of marks carried by each question is indicated at the end of the question.

(iii) The answer to each question or part thereof should begin on a fresh page.

(iv) Your answer should be precise and coherent.

(v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.

(vi) Candidates should attempt any **five** questions out of **twelve** questions.

(vii) If you encounter any typographical error, please read it as it appears in the text-book.

(viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.

(ix) No continuation sheets shall be provided to any candidate under any circumstances.

(x) Candidates shall put a cross (x) on blank pages of Answer Script.

(b) Verify Rolle's theorem for the function :

$$f(x) = \log\left[\frac{x^2 + ab}{x(a+b)}\right] \text{ in } [a, b],\ 0 \notin [a, b]. \quad 20$$

(c) Evaluate :

$$\lim_{x \to 0}\left[\frac{1}{x^2} - \cot^2 x\right]. \quad 20$$

4. (a) Assuming the possibility of expansion, show that

$$e^{ax}\sin bx = bx + abx^2 + \frac{b(3a^2 - b^2)}{3!}x^3 + \ldots\ldots \quad 20$$

(b) Find all the asymptotes of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0. \quad 20$$

(c) If u is a homogeneous function of degree "n" in x and y show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = nu.$ 20

5. (a) Trace the curve $r = a \sin 3\theta$. 20

(b) Find the area of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. 20

(c) Find the volume of the solid formed by revolving one arch of the cycloid, $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about the x-axis. 20

6. (a) Derive the equation of a plane passing through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3). 30

(b) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact. 30

7. (a) Find the equation of the ellipsoid with centre at origin, axes parallel to co-ordinate axes and passing through the points $(\sqrt{3}, 1, 1)$, $(1, \sqrt{3}, -1)$ and $(-1, -1, \sqrt{5})$. 30

(b) Find the generating lines of the hyperboloid of one sheet $\dfrac{x^2}{4} + y^2 - z^2 = 49$ through (10, 5, 1). 15

(c) Identify the intersection of the quadric surface $4x^2 + y^2 - \dfrac{z^2}{16} = 1$ with $y = \dfrac{\sqrt{3}}{2}$. Also find the foci and vertices. 15

8. (a) Solve :

$$\frac{dy}{dx} = \sin(x + y) + \cos(x + y). \quad 20$$

(b) Solve :

$$\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}. \quad 20$$

(c) Solve :

$$x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1. \quad 20$$

9. (a) Define exact differential equation. Show that the differential equation $Mdx + Ndy = 0$ is exact if and only if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ where M and N are functions of x and y. 20

(b) Solve :

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0. \quad 20$$

(xi) No blank page be left in between answer to various questions.

(xii) No programmable Calculator is allowed.

(xiii) No stencil (with different markings) is allowed.

1. (a) Show that the set $V = \{a + b\sqrt{2} \mid a, b \in Q\}$ is a vectorspace over the field of rationals under the operations of usual addition and scalar multiplication. 20

(b) If S and T are subspaces of a finite dimensional vectorspace V, show that dim S + dim T = dim $(S \cap T)$ + dim (S + T). 20

(c) If T is a mapping from $V_2(\mathbb{R})$ to $V_2(\mathbb{R})$ defined by $T(x, y) = (x \cos\theta - y \sin\theta, x \sin\theta + y \cos\theta)$ then show that "T" is a linear transformation. 20

2. (a) Find all eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. 20

(b) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}.$$ 20

(c) Determine whether $q(x, y, z) = x^2 + 2y^2 - 4xz - 4yz + 7z^2$ is a postive definite quadratic form. 20

3. (a) Discuss the differentiability of the function

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases} \text{ at } x = 0.$$ 20

(c) Solve :

$$(D^4 - 1)\, y = e^x \cos x \text{ where } D \equiv \frac{d}{dx}.$$ 20

10. (a) Prove that $\operatorname{div}(\vec{a} \times \vec{b}) = \vec{b} \bullet \operatorname{curl} \vec{a} - \vec{a} \bullet \operatorname{curl} \vec{b}$. 20

(b) Verify Stokes theorem for $\vec{f} = (2x - y)\, i - yz^2 j - y^2 zk$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C its boundary. 20

(c) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. 20

11. (a) A heavy uniform rod AB is hinged at A to a fixed point and rests in a position inclined at 60° to the horizontal, being acted upon by a horizontal force F applied at the lower point B. Find the action at the hinge and the magnitude of F. 20

(b) The base of an inclined plane is 4 m in length and height is 3 m. A force of 8 kg parallel to the plane, will just prevent a weight of 20 kg from sliding down. Find the coefficient of friction. 20

(c) A point moving in a straight line with simple harmonic motion has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is $2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$. 20

12. (a) A particle is projected with velocity u making an angle α with the horizontal. Find :

(i) the time of flight

(ii) the horizontal range

(iii) the greatest height attained. 30

(b) A solid right circular cone of vertical angle 60° is just immersed in water so that one generator is in the surface of the liquid. Prove that the resultant thrust on the curved surface of the cone to the weight of the water displaced by the cone is $\sqrt{7} : 2$. 30