Roll No. .... **Total No. of Pages : 5**

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Mathematics

(15)

Paper—II

Time : Three Hours] [Maximum Marks : 300

Note :— (i) Answers must be written in English.

(ii) Number of marks carried by each question are indicated at the end of the question.

(iii) Part/Parts of the same question must be answered together and should not be interposed between answers to other questions.

(iv) The answer to each question or part thereof should begin on a fresh page.

(v) Your answers should be precise and coherent.

(vi) Attempt **five** questions in all, selecting at least **two** questions from each of the Sections A & B.

(vii) If you encounter any typographical error, please read it as it appears in the text-book.

SECTION–A

1. (a) For a prime number P, prove every group G of order p^2 is abelian. 30

(b) Let G be an abelian group. Let H be the subset of G consisting of the Identity 'e' together with all elements of G of order 2. Show that M is a subgroup of G. 30

2. (a) (i) Show that a ring 'R' has no non-zero nilpotent element if and only if '0' is the only solution of $n^2 = 0$ in R.

(ii) In the ring Z_n, the divisors of '0' are precisely those non-zero elements that are not relatively Prime to n. 30

6. (a) Evaluate $\int_0^\infty \frac{\cos mx}{x^4+x^2+1}dx$. 30

(b) Show that the derivative of a function is analytic in a domain if itself an analytic function. 30

7. (a) Reduce the equation :

$$(n-1)^2\frac{\partial^2 z}{\partial x^2}-y^{2n}\frac{\partial^2 z}{\partial y^2}=ny^{2n-1}\frac{\partial z}{\partial y}$$

to canonical form and find its general solution. 30

(b) Find the solution of the equation :

$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=e^{-x}\cos y$, which tends to zero $x\to\infty$ and hence the value cos y when x = 0. 30

8. (a) Let f and g be arbitrary functions in their respective arguments, show that $u=f(x-\vartheta t+i\alpha y)+g(x-\vartheta t-i\alpha y)$ is a solution of

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{C^2}\frac{\partial^2 u}{\partial t^2}$ provided $\alpha^2=1-\frac{\vartheta^2}{C^2}$. 30

(b) Solve the partial differential equation $p^2q\,(x^2+y^2)=p^2+pq$ 30

SECTION—B

9. (a) (i) Prove that, in a simple dynamical system T+VC constant.

(ii) Discuss the motion of a particle in space using Cartesian co-ordinates. 30

(b) Construct the Lagrangian and the equations of motion of a coplanar double pendulum placed in a uniform gravitational field. 30

10. (a) (i) Find the curve for which the surface of revolution is minimum.

(ii) A uniform hoop is rolling down on an inclined plane without slipping. Find its motion. 30

(b) A particle moves under the influence of gravity on the frictionless inner surface of a cone $x^2+y^2=c^2z^2$. Obtain the equations of motion. 30

11. (a) (i) Derive the Navier-Stokes equation.

(ii) What is a Newtonian fluid ? 30

(b) Discuss about the flow near an infinite plate. 30

12. (a) Discuss about the flow past a cyclider. 30

(b) Derive the equation of continuity of fluid. 30

13. (a) What is an order of convergence ? Find the order of convergence of Regula-Falsi method. 30

(b) Write Newton-Raphson formula to obtain the $_4\sqrt{N}$ and also evaluate $_4\sqrt{32}$. 30

14. (a) Find the gradient of the road at the middle point of the elevation about a datum time of seven points of a road which are given below :

x	0	300	600	900	1200	1500	1800
y	135	149	157	183	201	205	193

30

(b) Solve $\frac{dy}{dx}=1-y$, y (0) = 0 using Euler's method. Find x = 0.1 and x = 0.2. Compare the result with results of the exact solution. 30

OR

(c) Evaluate $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta$ using Simpson's rule taking 6 equal intervals. 30

(b) Let F be a field of a quotients of D and let L be any field containing D. Then there exists a map $\psi : F \rightarrow L$ that gives an Isomorphism of F with a subfield of L such that $\psi(a) = a$ for $a \in D$. 30

3. (a) A function 'f' is integrable with respect to α on [a, b] if and only if for every $\in > 0$ there exists a partition P of [a, b] such that

$$U(P, f, \alpha) - L(P, f, \alpha) < \in.$$ 30

(b) (i) Show that the function :

$f(x, y) = 2x^4 - 3x^2y + y^2$ was neither a maximum nor a minimum at (0, 0)

(ii) Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ is invariant for change of rectangular axes. 30

4. (a) Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$, and $z = x + y$. 30

(b) Evaluate $I = \iiint_E (y^2z^2 + z^2x^2 + x^2y^2)\, dxdydz.$ 30

5. (a) (i) Show that the function $f(z) = \sqrt{|xy|}$ satisfies the Cauchy-Riemann equations at the point z = 0 but is not differentiable there .

(ii) Show that an analytic function with constant modulus is constant. 30

(b) If two functions f(z) and g(z) are analytic in a domain D bounded by a rectifiable Jordan Curve C and are continuous on C and $|g(z)| \leq |f(z)|$ for every point $z \in C$, then the two functions f(z) and f(z) + g(z) have the same number of zeros in D. 30

15. (a) Let X have the uniform distribution over the interval $(-\pi/2, \pi/2)$ show that y = tan x has a Cauchy distribution. 30

OR

(b) Let X have a binomial distribution with parameters n= 288 and P = 1/3. Use Chebyshev's inequality to determine a lower bound for Pr (76 < × < 116). 30

(c) Find the 25th percentile of the distribution having p.d.f.

$$f(x) = \begin{cases} \frac{|x|}{4} & \text{if } -2 < x < 2 \\ 0 & \text{elsewhere} \end{cases}.$$ 30

16. (a) Let the random variable x have a distribution of Probability about which we assume only that there is a finite variance σ^2. This of course, implies that there is a mean μ, then for every

$k > 0$ $Pr(|x - \mu| \geq k\sigma) \leq \frac{1}{k^2}$,

(or) equivalently

$$Pr(|x - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}.$$ 30

OR

(b) Let $f(x) = \begin{cases} \frac{x}{6}, & x = 1, 2, 3 \\ 0, & \text{else where} \end{cases}$

be the p.d.f of x. Find the distribution function and the p.d.f of $y = x^2$. 30

(c) An airline claims that only 6% of all lost luggage is never found. If, in a random sample 17 of 200 pieces of lost luggage are not found, test the null hypothesis P = 0.06 against the alternative hypothesis P > 0.06 at 0.05 level of significance. 30